Sequence 1 : Introduction to mathematical programming using GAMS

Unit 1.1 : Constrained optimization

Lesson 1 : An initial example

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Introduction

→ maximize
→ minimize

Optimize $Z = f(X)$

under the constraints $g_i(X) \leq 0$ for any given *i*

where *X* is the vector of the decision variables composed of j elements x

with $x_j \geq 0$ for any given j

f(*X*) is the objective function

F(X) linear → linear programming
F(X) non-linear → mathematical programming

Linear/mathematical programming

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Formulation and resolution of constrained optimization problems

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Linear programming :
Commonly used in the industrial sector
e.g. the animal feed industry

Introduction

€

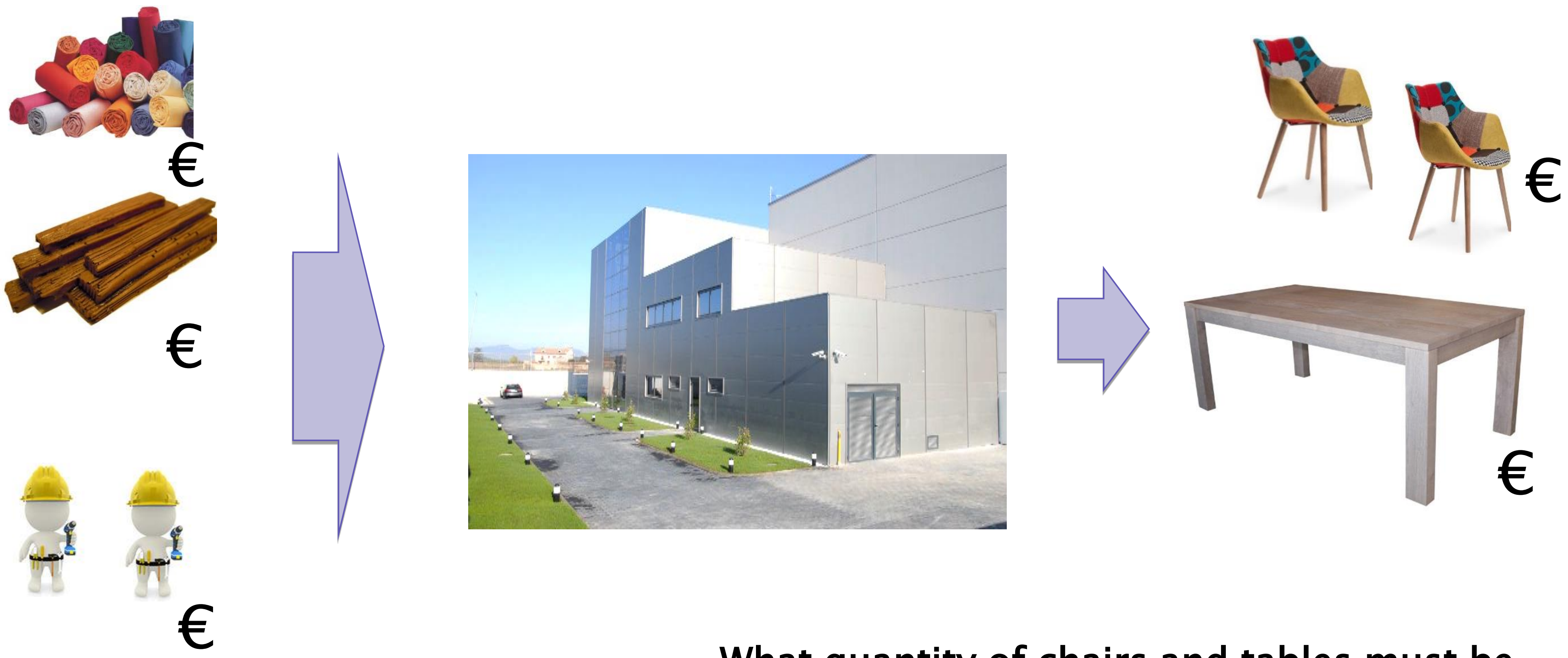
€

€

GUARANTEES	
Proteinaceous Material (min %)	14,5
Fat (min %)	1,6
Cellulosic Material (max %)	7,5
Mineral Matter (max %)	14
Vitamins (% kg) A.UL.	1.000.000
Vitamins (% kg) D3.U.I.	200.000

What is the optimal composition that meets the expected contents while minimizing production costs ?

Introduction



What quantity of chairs and tables must be produced in order to maximize the company income ?

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A cereal farm

A given farmer has land and labour and can grow both wheat and maize. The optimal area dedicated to wheat and maize cultivation must be determined, knowing that :

- The objective of the farmer is to maximize his net income (or gross margin)

- Wheat yields 450€ and maize 1000€ per ha

- Wheat requires 25h of labour and maize 50h per ha

- The total farm area is 50 ha and the farmer can only work 2000 h a year

Pause the video and try to formulate this problem, then resume the video and watch the solution

General formulation, algebraic writing

Maxi $Z = 450\ X_1 + 1000\ X_2$

subject to

$$X_1 + X_2 \leq 50$$

$$25\ X_1 + 50\ X_2 \leq 2000$$

$$X_1 \geq 0\ ;\ X_2 \geq 0$$

where
- Z value of the objective function
- x_j level of activity or decision variables
- c_j economic returns for each activity
- a_{ij} matrix of technical coefficients
- b_i resource availability

where
- Z farm income (euros)
- X_1 area dedicated to wheat (ha) 
- X_2 area dedicated to maize (ha) 

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Mathematical formulation

1. Identifying and naming the variables
2. Writing the constraints
3. Writing the objective function

Z farm income (euros)
X_1 area dedicated to wheat (ha) 
X_2 area dedicated to maize (ha) 

Maximize $Z = 450\ X_1 + 1000\ X_2$ (€)

Subject to $X_1 + X_2 \leq 50$ (ha)

$25\ X_1 + 50\ X_2 \leq 2000$ (hours)

$X_1 \geq 0\ ;\ X_2 \geq 0$

General formulation, matrix writing

Maximiser $Z = c\,X$

where X is the vector of x_j
C is the vector of c_j
A is the matrix of a_{ij}
B is the vector of b_j

In our example


X vector composed of 2 elements $\begin{pmatrix} x_1 & x_2 \end{pmatrix}$

c economic yields vector $\begin{pmatrix} 450 & 1000 \end{pmatrix}$

b availability vector $\begin{pmatrix} 50 \\ 2000 \end{pmatrix}$

A matrix of technical coefficients $\begin{pmatrix} 1 & 1 \\ 25 & 50 \end{pmatrix}$

$$A\,X \leq B \Leftrightarrow \begin{pmatrix} 1 & 1 \\ 25 & 50 \end{pmatrix} * \begin{pmatrix} x_1 & x_2 \end{pmatrix} \leq \begin{pmatrix} 50 \\ 2000 \end{pmatrix} \Leftrightarrow \begin{matrix} x_1 + x_2 \leq 50 \\ 25x_1 + 50x_2 \leq 2000 \end{matrix}$$

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