

Sequence 1 : Introduction to linear programming using  
GAMS

Unit 1 : Constrained optimization

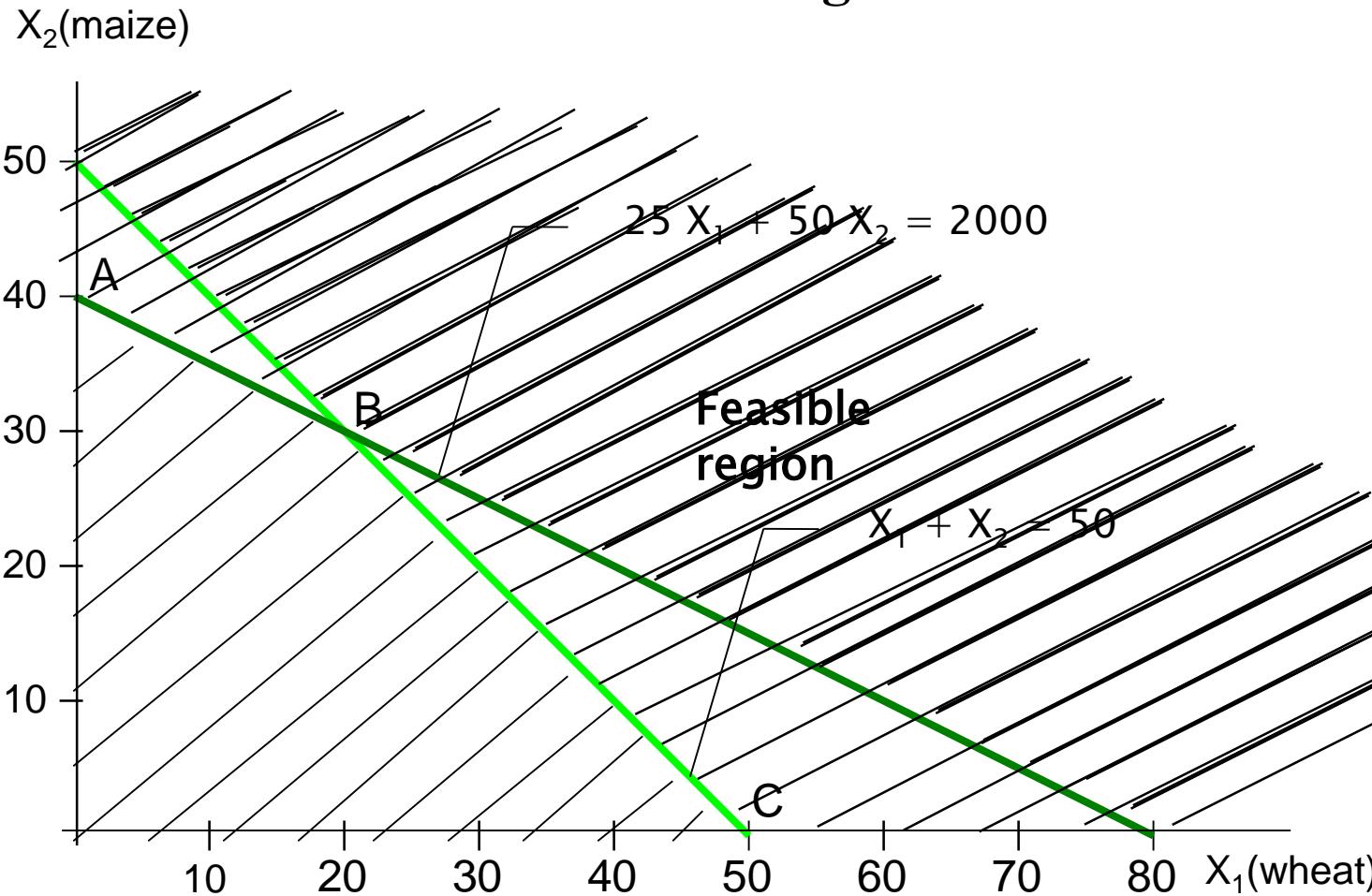
## Lesson 3 : One solution, several solutions, no solution ?

Florence Jacquet

ModelEco

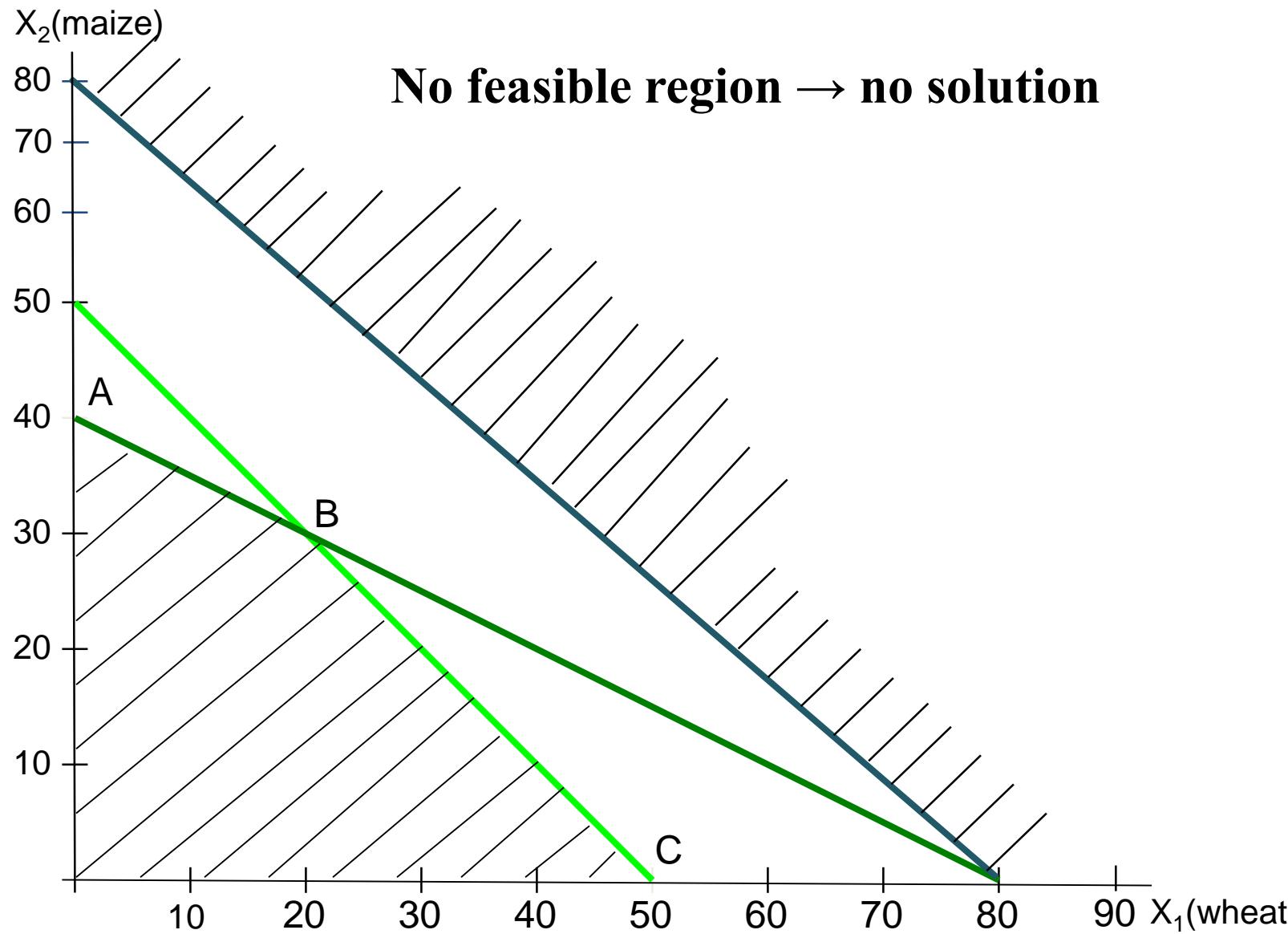
## Unbounded problem

**Unbounded feasible region → no solution**



Max  
subject to

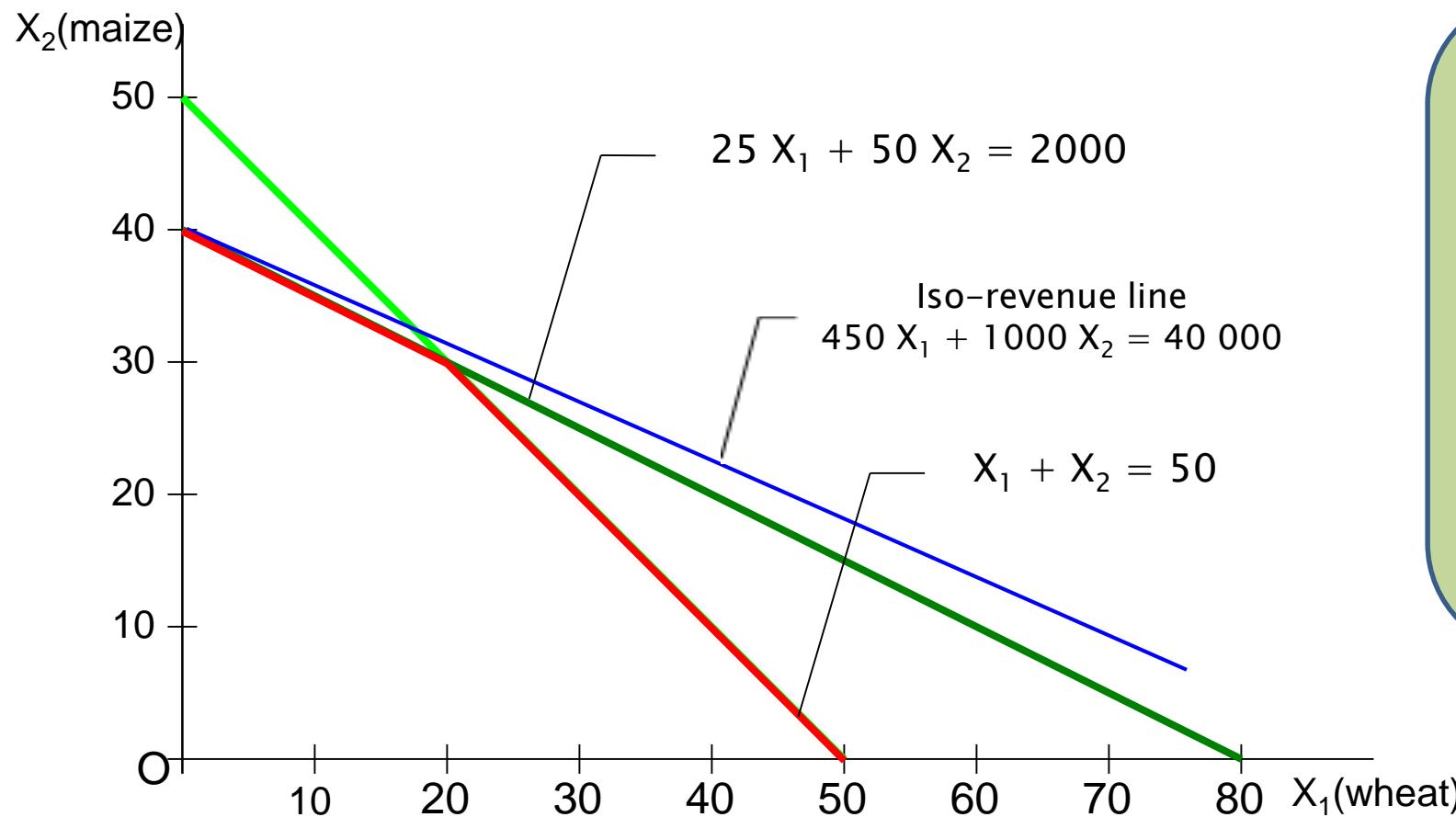
$$\begin{aligned}
 & Z = 450 X_1 + 1000 X_2 \\
 & X_1 + X_2 \geq 50 \\
 & 25 X_1 + 50 X_2 \geq 2000 \\
 & X_1 \geq 0 ; X_2 \geq 0
 \end{aligned}$$



## Infeasible problem

Maximiser  $Z = 450 X_1 + 1000 X_2$   
 subject to  
 $X_1 + X_2 \leq 50$   
 $25 X_1 + 50 X_2 \leq 2000$   
 $X_1 + X_2 \geq 80$   
 $X_1 \geq 0 ; X_2 \geq 0$

## Change in the Objective Function slope (1)



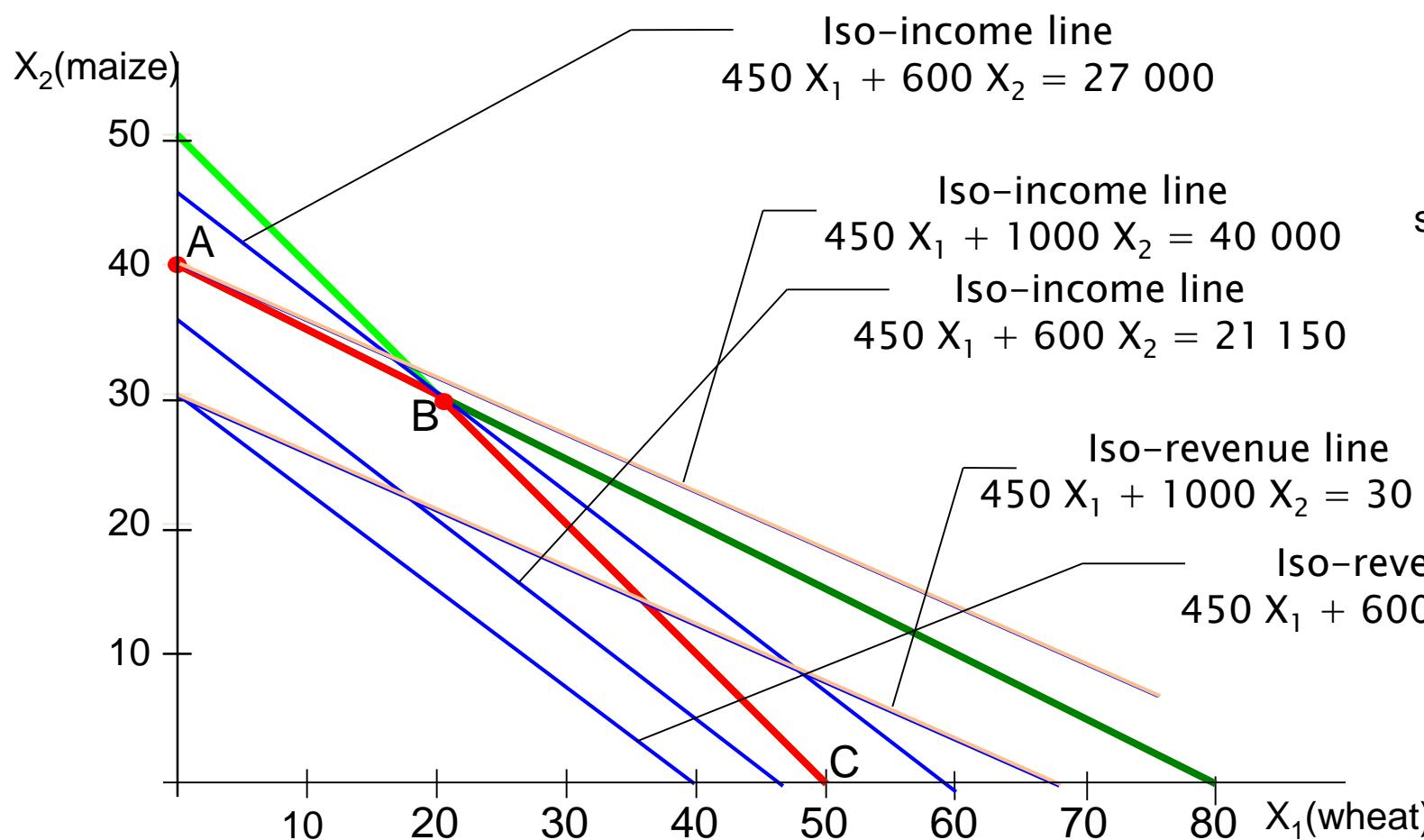
The gross margin of maize goes from 1000 to 600 euros

- What happens graphically ?
- The land constraint changes
  - The labour constraint changes
  - The iso-income lines change

Solution

$$\begin{aligned} X_1 &= 20 \\ X_2 &= 30 \\ Z &= 27000 \end{aligned}$$

## Change in the Objective Function slope (2)



Max

subject to

$$Z = 450 X_1 + \textcolor{red}{600} X_2$$

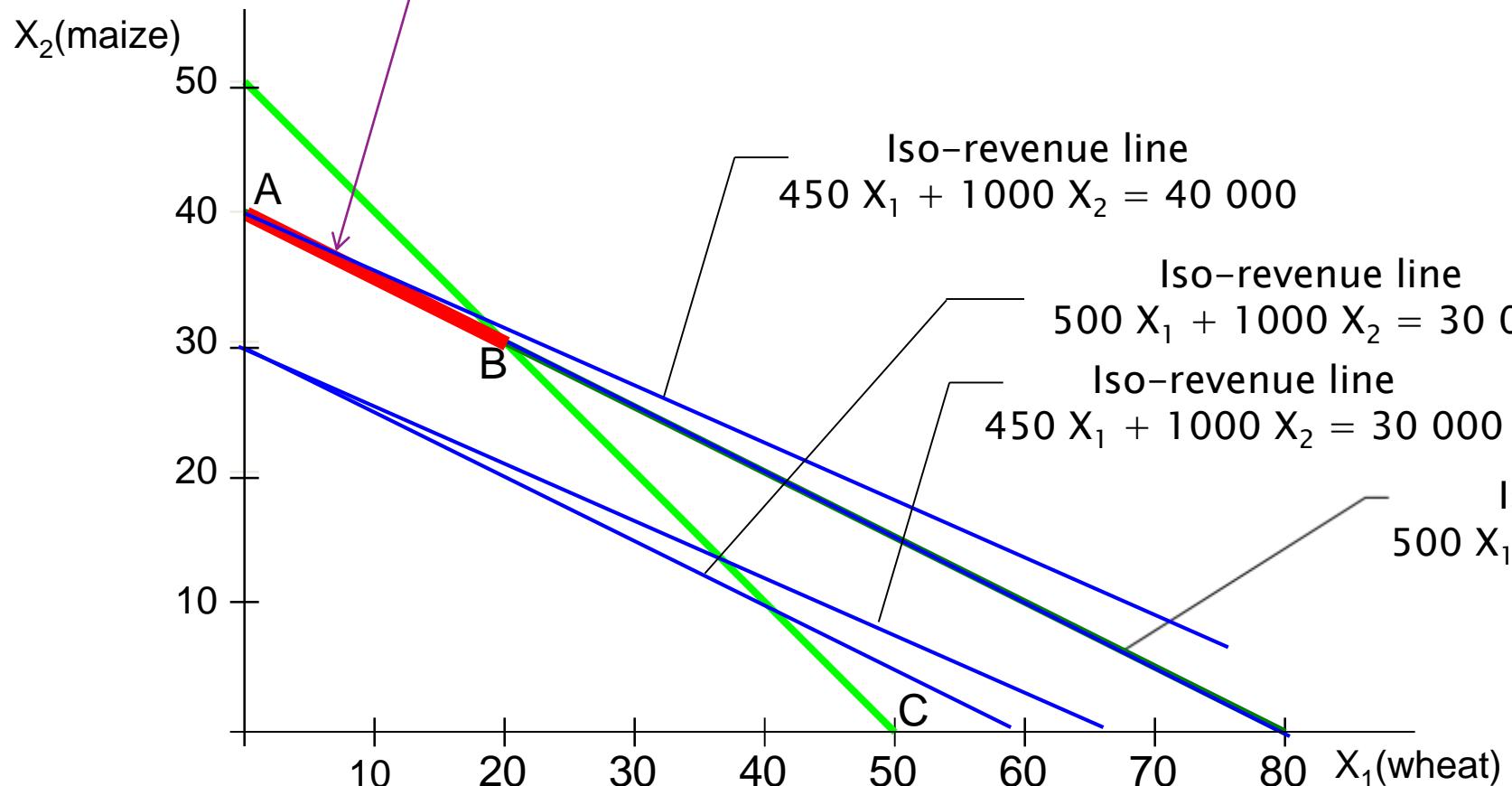
$$X_1 + X_2 \leq 50$$

$$25 X_1 + 50 X_2 \leq 2000$$

$$X_1 \geq 0 ; X_2 \geq 0$$

## Multiple Solution

**Degenerate solution**



Max

subject to

$$Z = 500 X_1 + 1000 X_2$$

$$X_1 + X_2 \leq 50$$

$$25 X_1 + 50 X_2 \leq 2000$$

$$X_1 \geq 0 ; X_2 \geq 0$$