Sequence 1 : Introduction to mathematical programming using GAMS

Unit 1.3 : Primal problem, dual problem

Lesson 9 : What are dual values ?

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Marginal values

Dual value ↓

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Objective :        40000.000000
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	LOWER	LEVEL	UPPER	MARGINAL
---- EQU OBJECTIVE	.	.	.	-1.000
---- EQU LAND	-INF	40.000	50.000	.
---- EQU LABOUR	-INF	2000.000	2000.000	20.000

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OBJECTIVE  objective function
LAND  land equation
LABOUR  labour equation
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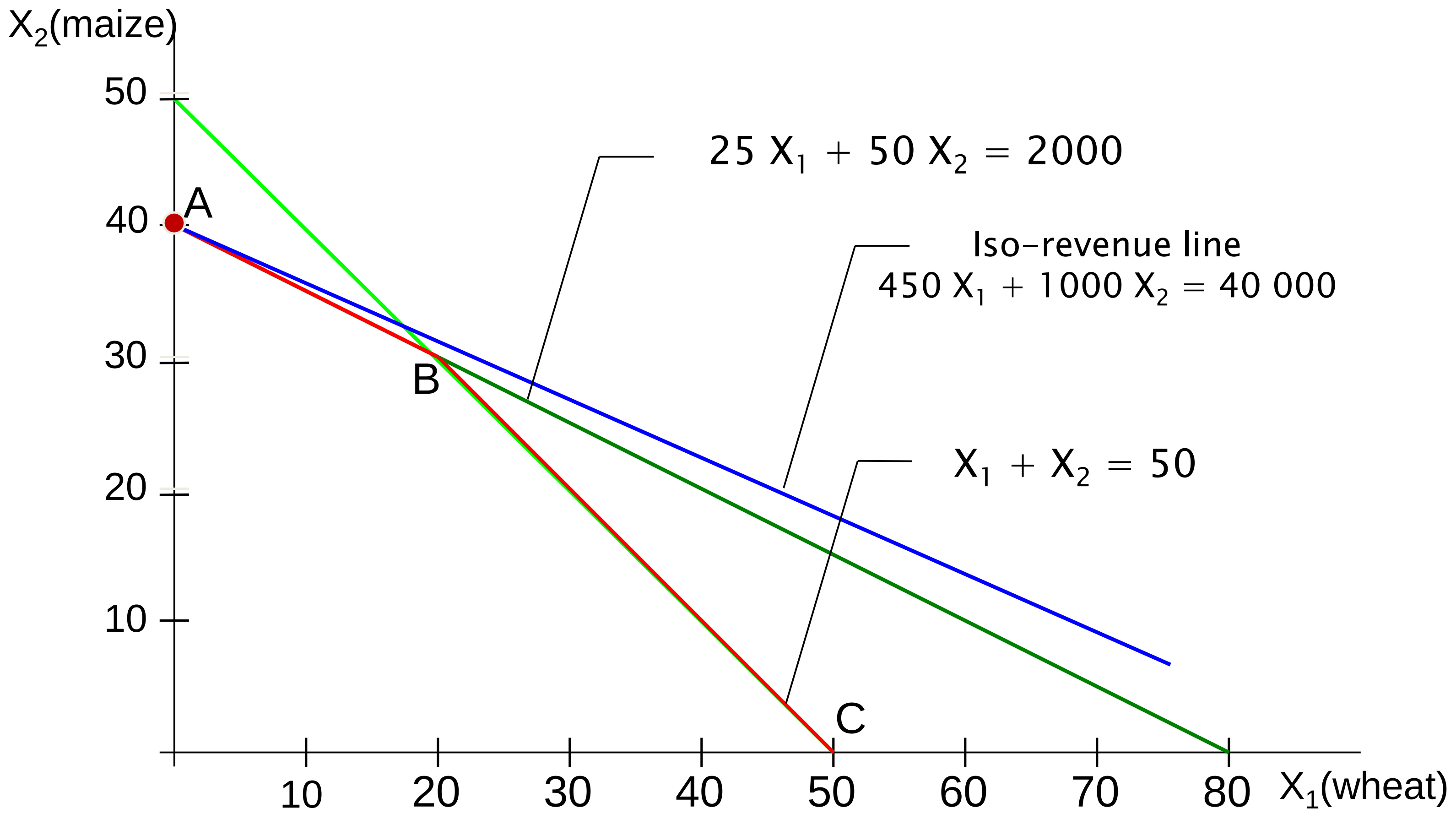
Vocabulary :
Binding constraint → A constraint which takes on its maximum value

+1h ⇒ +20€ therefore 40020€

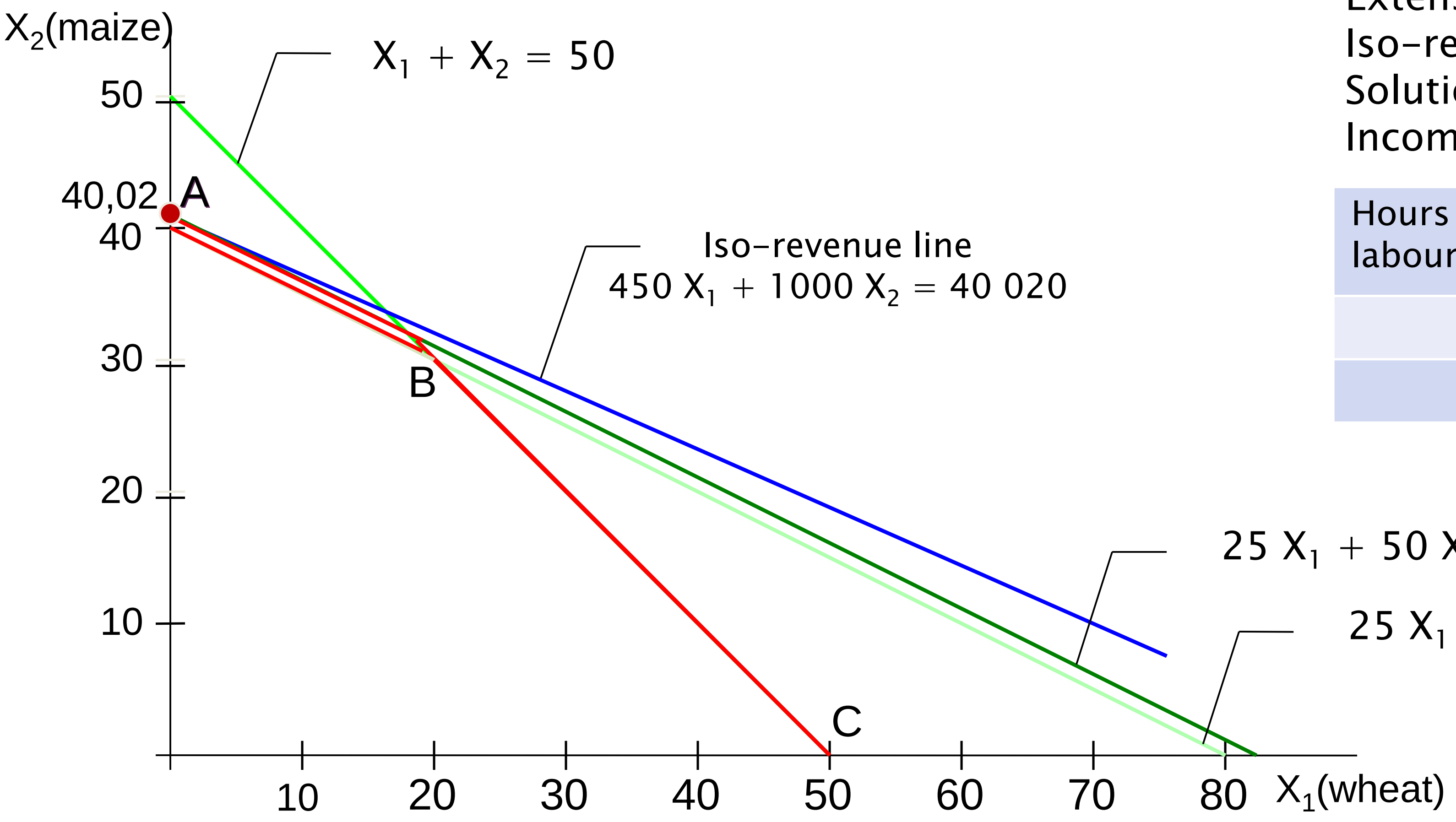
2000h ⇒ solution : 40000€

Initial Model



Solution A : $X_1=0$; $X_2=40$
Income : 40 000
The labour constraint is binding
The marginal value of labour is 20

Simulation of an increase by one hour of labour



Extension of the feasible region
Iso-revenue line moved
Solution A_2 : $X_1=0$; $X_2=40{,}02$
Income : 40 020

Hours of labour	For … ha of maize	Yields …
50 →	1 →	1000€
1 →	0,02 →	20€

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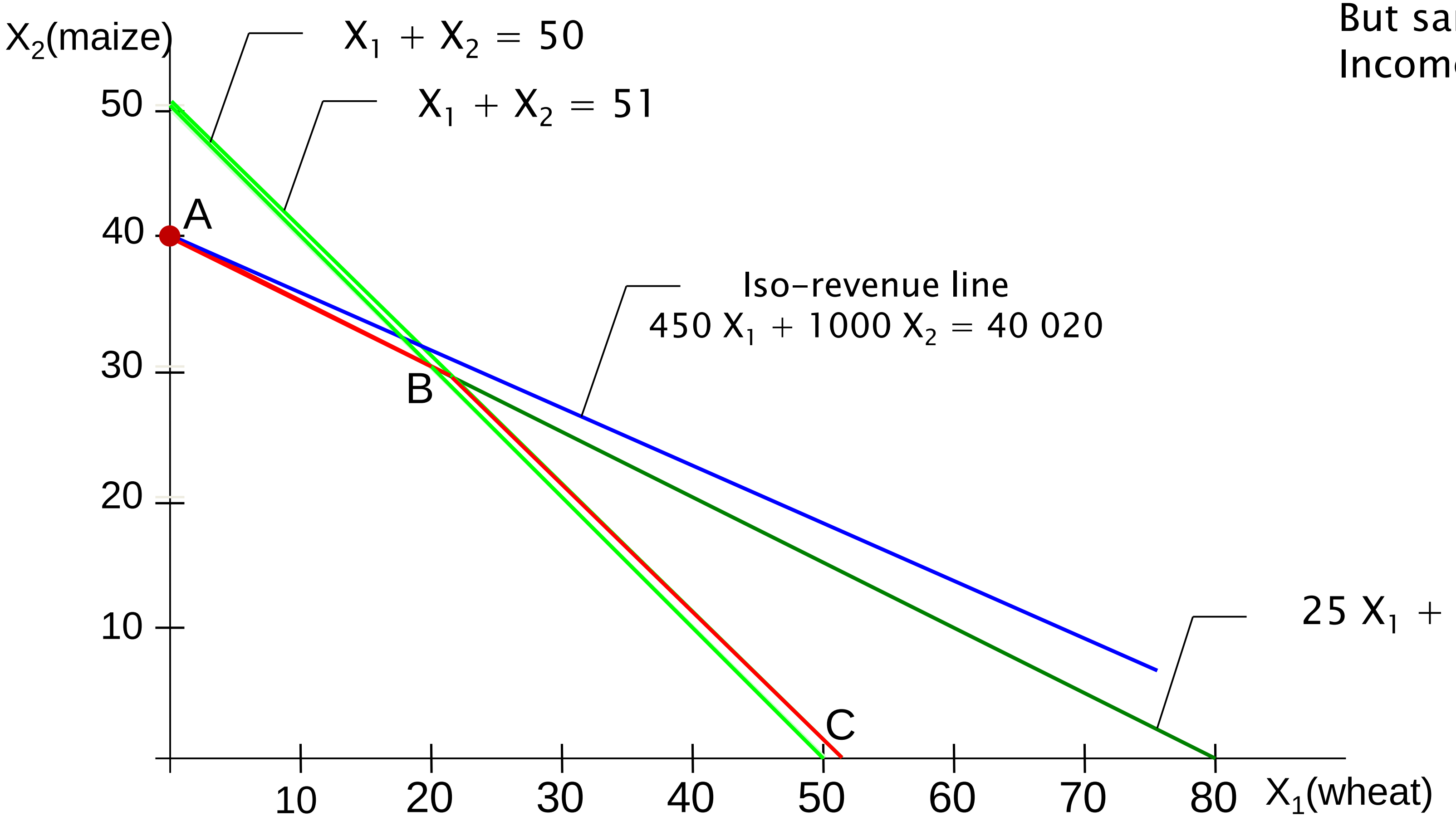
Simulation of an increase by one hectare of land



Extension of the feasible region
But same solution → A : $X_1=0$; $X_2=40$
Income : 40 000

In short...

- Every binding constraint is associated with a dual value of the resource in question.

- The dual values of non-binding constraints are equal to zero.

General formulation of a dual problem

Primal problem

Max $\underset{(1,n)}{Z = c} \; \underset{(n,1)}{x}$

Dual problem

Min $Z = \underset{(1,m)}{b'} \; \underset{(m,1)}{y}$

With $\underset{(n,m)}{A'} \; \underset{(m,1)}{y} \leq \underset{(n,1)}{c'}$

$\underset{(m,l)}{y} \geq 0$

A' transposed from A
b' transposed from b
c' transposed from c

$$cx^* = b'y^*$$

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Application at farm level

Primal problem	Dual problem
Max $Z = 450\, X_1 + 1000\, X_2$	Min $Z = 50Y_1 + 2000Y_2$
subject to $X_1 + X_2 \le 50$	subject to $Y_1 + 25Y_2 \ge 450$
$25\, X_1 + 50\, X_2 \le 2000$	$Y_1 + 50Y_2 \ge 1000$
$X_1 \ge 0\, ;\, X_2 \ge 0$	$Y_1 \ge 0; Y_2 \ge 0$

Dual values = Opportunity costs = Implicit values = « shadow prices »

x1 : area dedicated to wheat (h

x2 : area dedicated to maize (ha

x1=0; x2*=40; Z=40000*

y_1 : *land opportunity cost (€)*

y_2 : *labour opportunity cost (€)*

y_1*=0; y_2*=20; y_3*=0; Z= 40000

Economic interpretations

Primal problem	Dual problem
Maximization of an income under availability and resource constraints	Minimization of the total cost of resources under the constraint of their contribution to the income of productions
→ Calculates the quantities/areas for each activity	→ Calculates the prices for each resource

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