Sequence 3 : Modelling risk in agricultural economics

Unit 1 : Agriculture, a risky activity

Lesson 23 : Expectation / Standard deviation

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Modelling behaviour in the face of risk – Different approaches

- Taking risk into account using a security constraint

- Minimization of income variability or maximization of the expected utility of income

Minimization of the standard deviation and downward restriction of expectation

$$\text{Min } \sigma(\tilde{Z}) \quad \text{with} \quad \mathbb{E}(\tilde{Z}) \geq Z_0$$

Maximization of expectation and upward restriction of variance

$$\text{Max } \mathbb{E}(\tilde{Z}) \quad \text{with} \quad \sigma(\tilde{Z}) \leq \beta$$

$$\text{Max } (\mathbb{E}(\tilde{Z}) - \phi\, \sigma(\tilde{Z}))$$

Maximization of expectation and minimization of variance

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Expectation-Standard deviation approach

Equations

Objective function

Maximize $F=\mathbb{E}(\tilde{Z})-\phi.\sigma(\tilde{Z})$

Inequality

Under constraints $\sum_j a_{ij} x_j \leq b_i \quad \forall i$

$x_j \geq 0 \quad \forall j$

Equality

With $Z_s=\sum_j c_{js} x_j \quad \forall s$

$\mathbb{E}(\tilde{Z})=\sum_s [p_s Z_s]$

$\sigma(\tilde{Z})=\sqrt{[\sum_s Z_s - \mathbb{E}(\tilde{Z})]^2 . p_s}$

F	function to maximize
ϕ	risk aversion weighting coefficient if $\phi = 0$ then indifferent to risk increases with risk aversion
s	states of nature
c_{js}	income of activity j in state of nature e
$\mathbb{E}(\tilde{Z})$	income expectation
$\sigma(\tilde{Z})$	income standard deviation
p_s	probability of state e

In GAMS

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In GAMS – Square root and square
Cereal farm example

$$\mathbb{E}(\tilde{Z})\text{-}\phi.\sigma(\tilde{Z})$$

$$Z_s=\sum_j c_{js}x_j \qquad \forall\ s$$

$$\mathbb{E}(\tilde{Z})= \sum_s [p_s\ Z_s]$$

$$\sigma(\tilde{Z})=\sqrt{[\sum_s Z_s - \mathbb{E}(\tilde{Z})]^2.\ p_s}$$

GAMS notation :

SQR	Square
SQRT	Square root

With :

S	states of nature (set)	
p(S)	probability of S	(parameter)
GM(C,S)	gross margin for every C according to the state (table)	
X(C)	variable, optimum number of C (variable)	

- Random income calculation

```
randomIncome(S).. RI(S) =e= sum(C, GM(S,C)*X(C));
```

- Income expectation calculation

```
income.. GM =e= sum(S, RI(S)*p(S));
```

- Standard deviation calculation

```
standardeviation..  SD =e= SQRT(sum[E, SQR(RI(S)-GM)*p(S)]);
```

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$$\sigma(\tilde{Z})=\sqrt{}\ [\sum_s Z_s - \mathbb{E}(\tilde{Z})]^2 .\ p_s$$

$$\sigma(\tilde{Z})=\sqrt{}\ [\sum_s Z_s{}^2 + \mathbb{E}(\tilde{Z})^2 - 2.\ Z_s - \mathbb{E}(\tilde{Z})].\ p_s$$

$$\sigma(\tilde{Z})=\sqrt{}\ [\sum_j c_{js} x_j + \mathbb{E}(\tilde{Z})^2 - 2.\ Z_s - \mathbb{E}(\tilde{Z})].\ p_s$$

Pause the slideshow, download the risk_base.gms model and add the risk with the Expectation-Standard deviation method.
Take your time !

Modification of the Run command

```
solve modeleRisk using NLP maximizing F ;
```

Type of model	LP	NLP	MIP	MINLP
Meaning	Linear programming	Nonlinear programming	Mixed integer programming	Mixed integer Nonlinear programming
Use	For linear models	For non-linear models (case of a variable multiplied by itself or by another variable)	Model with integer and linear variables	

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