Sequence 3 : Risk in agricultural economics models

Unit 1 : Agriculture, a risky activity

Lesson 24 : Chance-constrained models

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Behaviour in the face of risk as an income security constraint

- Safety First $Z_e \geq Z_0 \quad \forall$ e, state of nature

$Pr\{\tilde{Z} \leq Z_0\} \leq \alpha$ if $\alpha = 0 \quad \Leftrightarrow \quad Z_s \geq Z_0$

- General formulation Chance Constrained Model (Charnes, Cooper 1959)

$Pr\{\tilde{Z} \leq Z_0\} \leq \alpha$

$\tilde{Z}$ random income
Z_0 minimum income wanted by the farmer
α tolerated probability that the minimum income level may not be reached

- Linearized formulation : Target-MOTAD (Tauer, 1983)

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Chance-Constrained Model

If there is a sufficient number of states of nature and $\tilde{Z}$ follows a normal distribution, then :

$\Pr\{\tilde{Z} \leq Z_0\} \leq \alpha$ is written $\mathbb{E}(\tilde{Z}) - Z_0 > \theta_\alpha \, \sigma(\tilde{Z})$

With θ_α value from Laplace Gauss tables

Data from Laplace Gauss tables					
α	0,3	0,2	0,1	**0,05**	0,01
θ	0,53	0,84	1,28	**1,65**	2,33

For example, if the farmer wants to be 95% certain that he will receive an income of 18000, then :

$\Pr\{\tilde{Z} \leq 18000\} \leq 0{,}05$ is written $\mathbb{E}(\tilde{Z}) - 18000 > 1{,}65 \, \sigma(\tilde{Z})$

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Writing in GAMS

GAMS notation :

SQR	Square
SQRT	Square root

With :

AI	income expectation (variable)
S	states of nature (set)
Z0	minimum income scalar (scalar)
teta	probability function coef. (1,65 pour 95%) (scalar)
p(S)	probability of S (parameter)

Max $\mathbb{E}(\tilde{Z})$

s.t. $\mathbb{E}(\tilde{Z}) - Z_0 > \theta_\alpha \sigma(Zs)$

$\mathbb{E}(\tilde{Z}) = \sum_s [p_s \sum_j c_{js} x_j]$

$\sigma(\tilde{Z}) = \sqrt{\sum_s [Z_s - \mathbb{E}(\tilde{Z})]^2 . p_s}$

⇓

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objective..            RM =e= sum(S, RI(S)*p(S));
CCM..                  AI-Z0 =g= teta*SD;
randomIncome(S)..       RI(S) =e= sum(C, GM(S,C)*X(C));
standardDeviation.. SD =e= SQRT(sum[S, SQR(RI(S)-Z)*p(S)]);
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Linearization of the chance-constrained model

Large model case, several non-linearities : Linearizing to facilitate resolution
Target-MOTAD : Variation of MOTAD with chance-constrained linearization
Target = minimum income wanted

Maximize $\mathbb{E}(\tilde{Z}) = \sum_s [\, p_s \sum_j c_{js} x_j]$

Under the constraints $\sum_j a_{ij} x_j \leq b_i \quad \forall i$

$$\sum_j c_{js} x_j \geq Z_0$$

$$\sum_s p_s D_s \leq \lambda$$

$$x_j, D_s \geq 0 \quad \forall j, s$$

Z0 : threshold income
D_s : total of all income deviations below the threshold income
p_s : occurrence probability of state of nature e
λ : total deviation tolerance level in relation to minimum income Z0

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Writing in GAMS

$$Z_0 - D_s \leq \sum_j c_{js} x_j \qquad \forall s$$

$$\sum_s p_s D_s \leq \lambda$$

- Deviation from the minimum income for each state of nature

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deviation(S).. Z0-DEV(S) =l= RI(S) ;
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- Constraint limiting the weighted sum of deviations

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lambda.. sum(S,DEV(S)*p(S)) =l= LBD ;
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With :

DEV(S) positive variable, measures the deviation above Z0 per state of nature

LBD scalar that measures the tolerated average deviation for example chosen according to the characteristics of the farmer

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