Sequence 3 : Modelling risk and time

Unit 1 : Risk

Lesson 25 : Reminder of the expected utility function

Florence Jacquet

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Risk aversion

L1 : we have a 50% chance of earning 1000 and a 50% chance of not earning anything

L2 : we earn 500 each time

States of nature	Probabilités	Gains liés à L1	Gains liés à L2
S1	0.5	1000	500
S2	0.5	0	500
Espérance de gain		500 =	500

Risk-averse

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The certainty equivalent notion

States of nature	Probabilities	Gains linked to L1	Gains linked to L2
S1	0.5	1000	500
S2	0.5	0	500
Gain expectation		500	500

Certainty equivalent :

« Certain amount » which provides the same utility

Which amount would be equivalent to L1, according to you ?

- ❑ 0
- ❑ 100
- ❑ 200
- ❑ 300
- ❑ 400
- ❑ 500
- ❑ 600
- ❑ 700

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Risk premium

Sates of nature	Probabilities	Gains linked to L1	Gains linked to L2
S1	0.5	1000	500
S2	0.5	0	500
Gain expectation		500	500

Certainty equivalent :
« Certain amount » which provides the same utility

Certainty equivalent of 500 : risk-neutral

Certainty equivalent ≤ 500 : risk-averse

Risk premium = gain expectation – certainty equivalent

Example : For the risk-averse individual

$EC(L_1)=200$

$PR=500-200=300$

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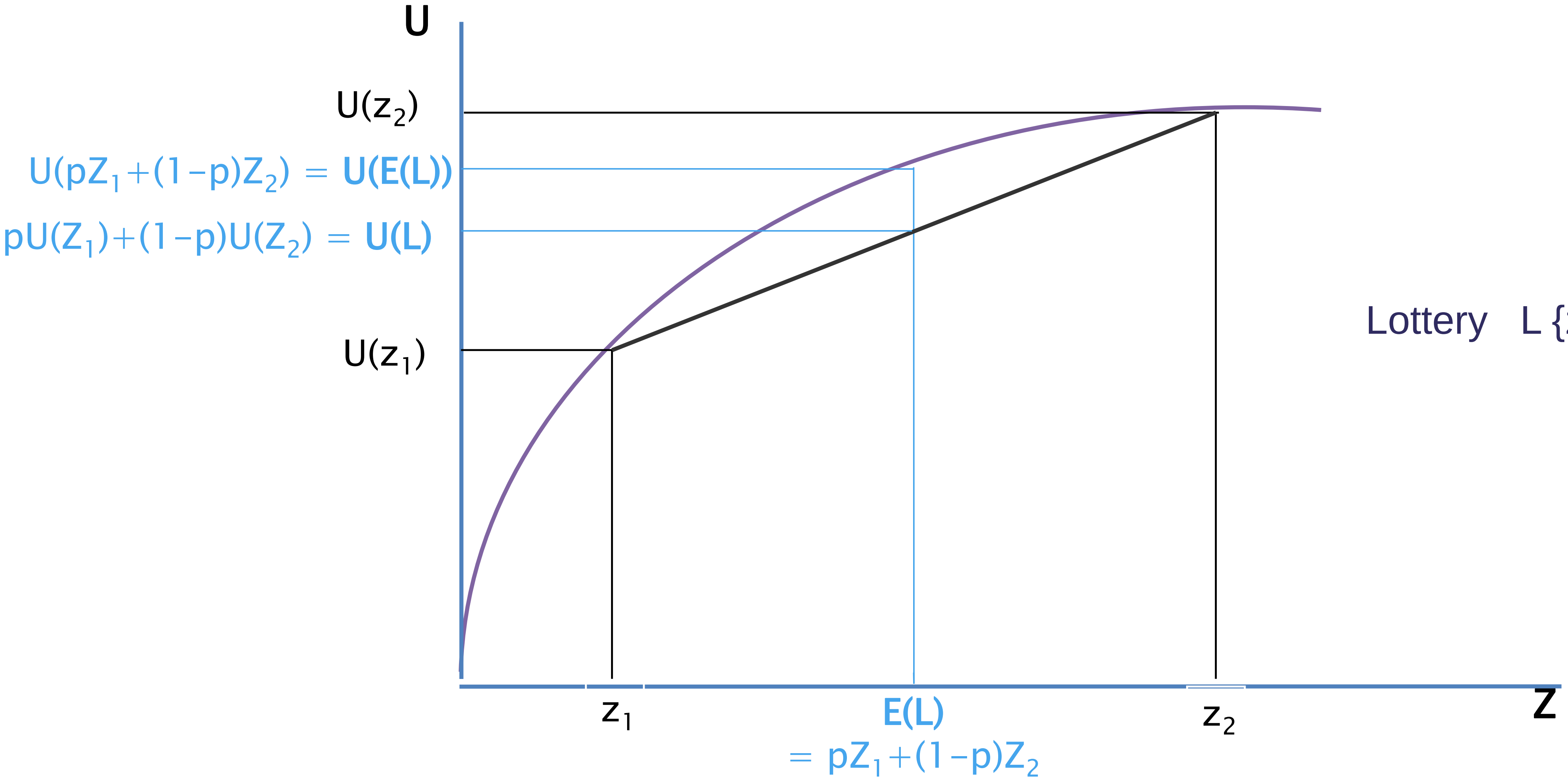
Expected utility theory

- Suggested by by Bernouilli (1738), developed by von Neumann and Morgenstern (1944)
- The expectation of the utility of gains supplants the expectation of gains

$$\Sigma\, p_i x_i \rightarrow \Sigma(\, p_i\, U(x_i)\,)$$

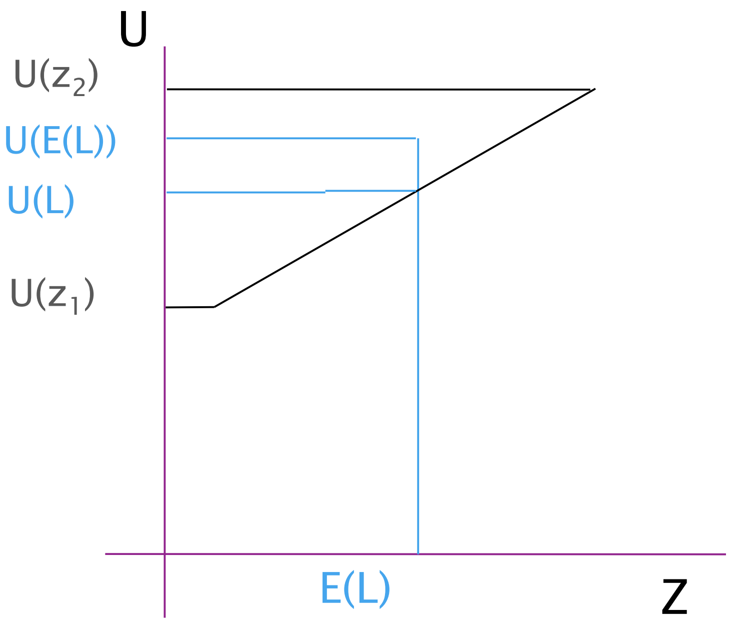
Expected utility theory



Lottery $L\ \{z_1, z_2;\ p, 1\text{-}p\}$

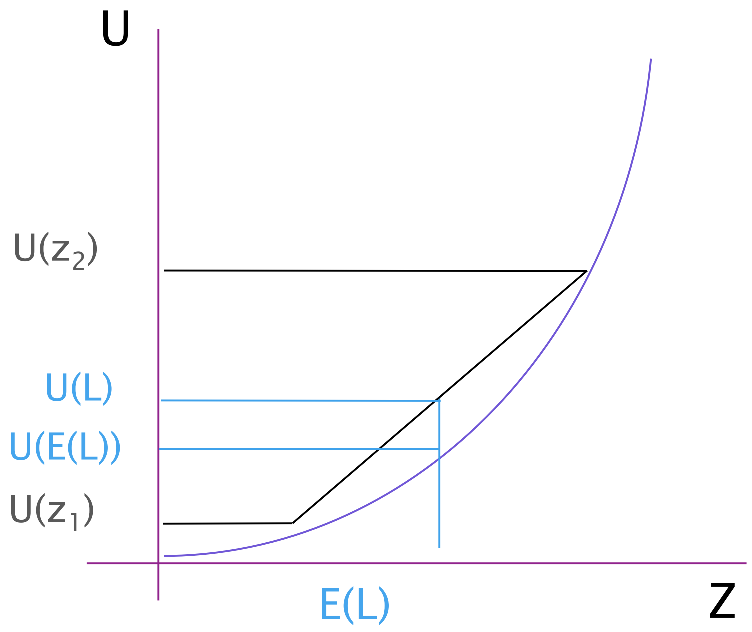
Expected utility theory

Concave function



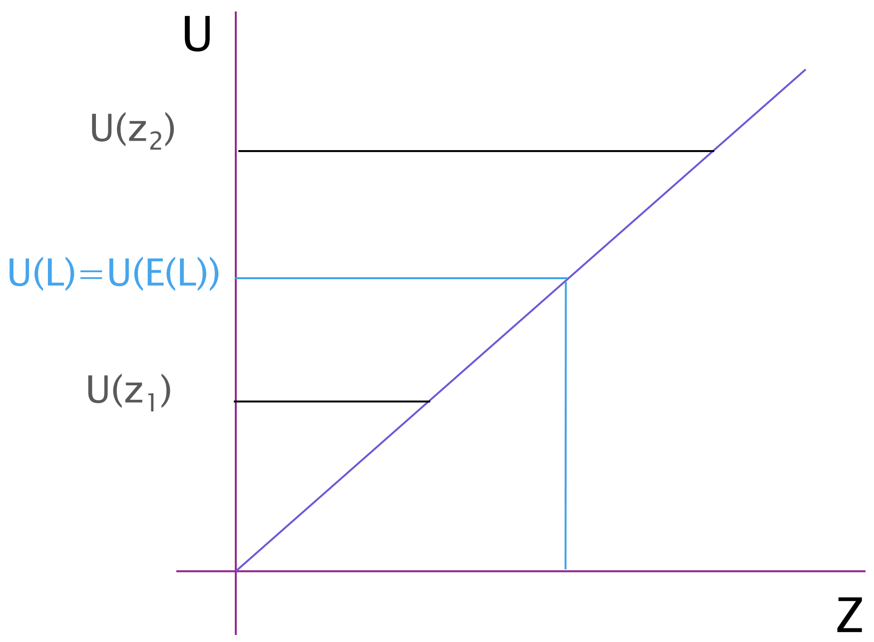
Risk-averse
$U(L)>U(E(L))$

Convex function



Risk-prone
$U(L)>U(E(L))$

Linear function

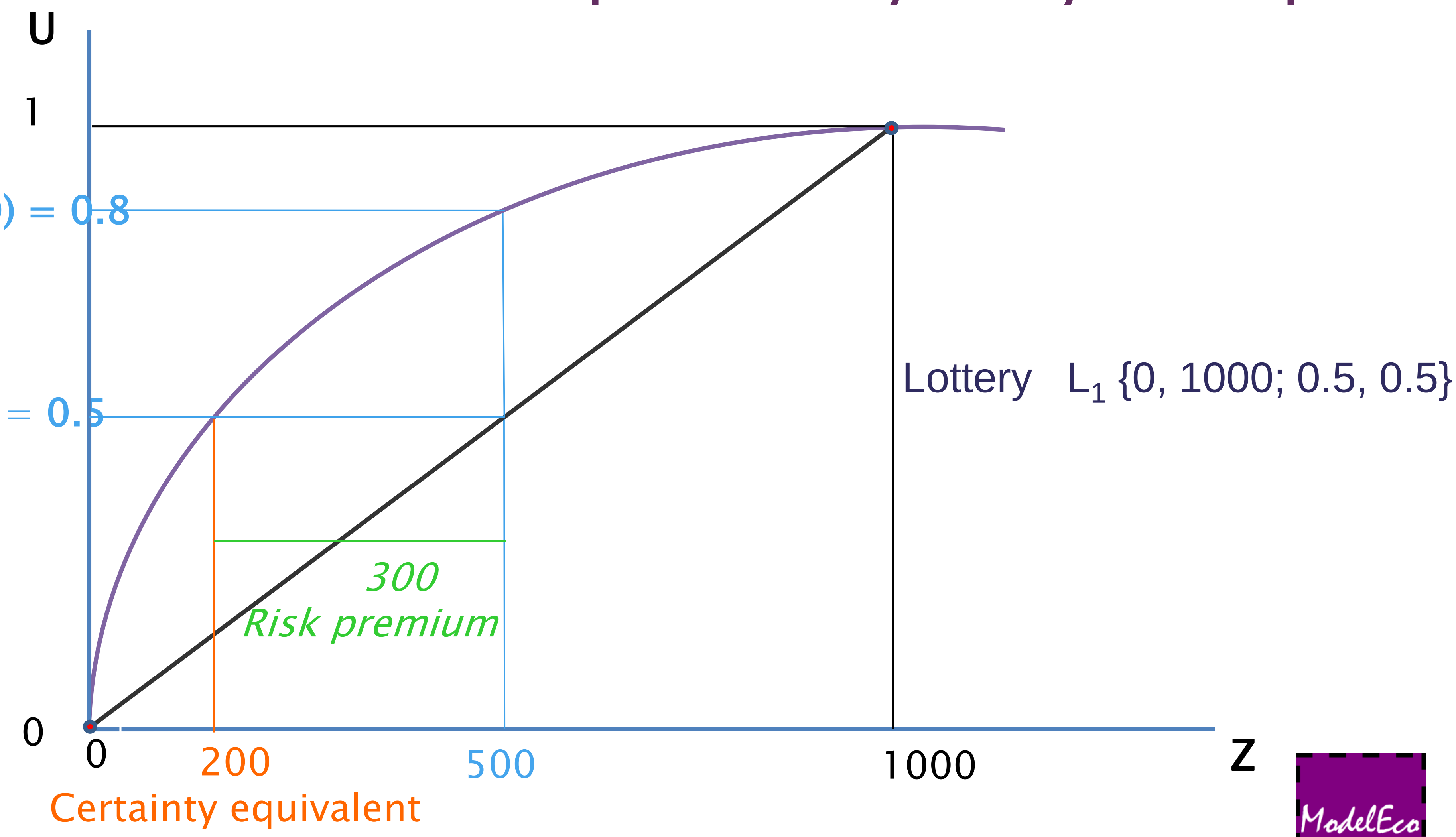


Risk-neutral
$U(L)=U(E(L))$

Expected utility theory – example

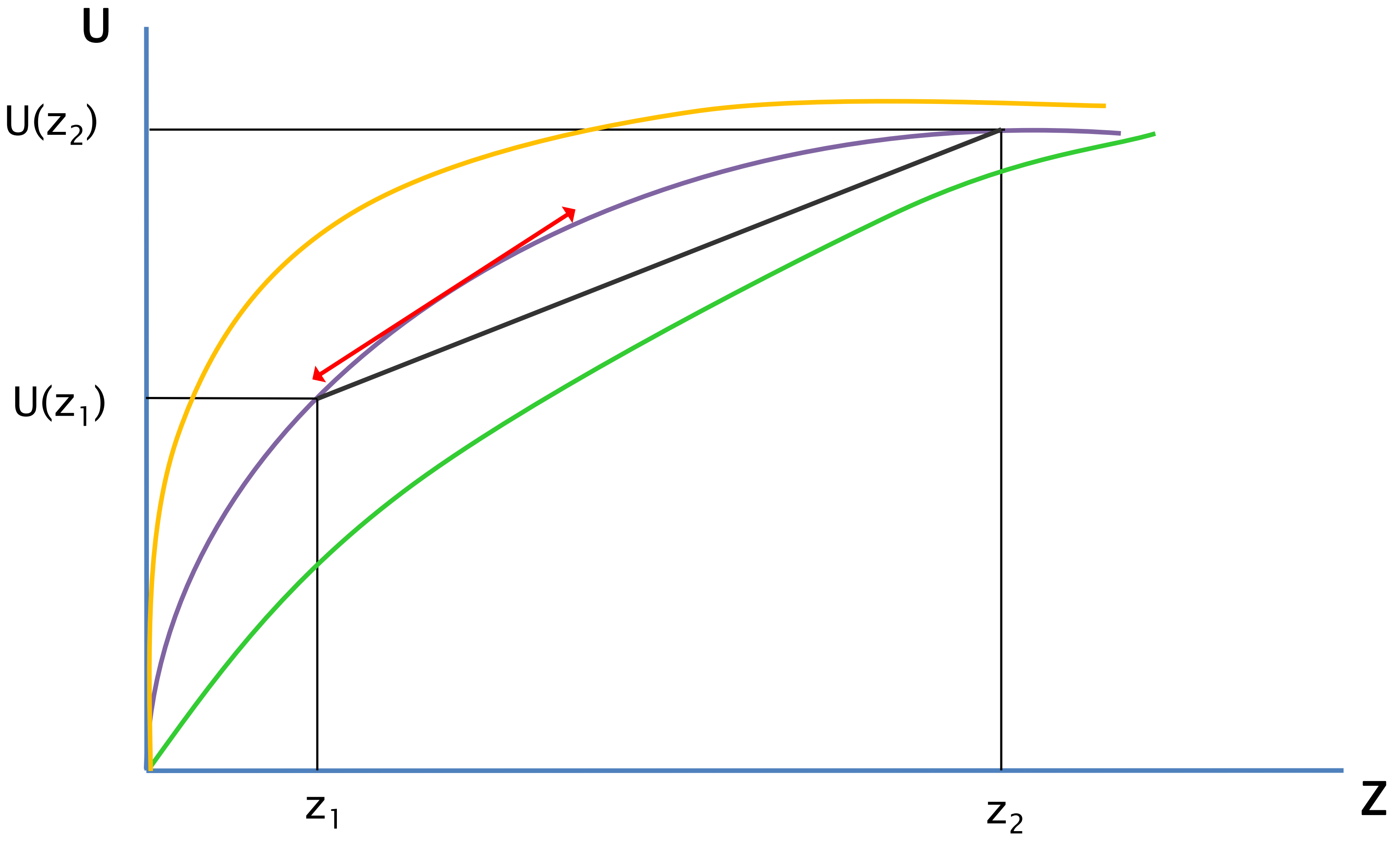
U(0.5*1000+0.5*0) = **U(500) = 0.8**

$U(L_1)$ = 0.5 U(1000)+0.5U(0) = **0.5**



Arrow Pratt coefficients

U' positive ; U'' negative



Choosing a utility function

Max U(Z)

→ Functional forms :

With a constant absolute risk aversion (CARA) :

- $U(Z) = 1 - e^{-aZ}$, where a is the absolute risk aversion coefficient

we have $U' = ae^{-aZ}$; $U'' = -a^2e^{-aZ}$; ARA = a ; ARR = aZ

With a decreasing absolute risk aversion (DARA) :

- $U(Z) = Z^a$ with $0 < a < 1$

we have $U' = aZ^{a-1}$; $U'' = a(a-1)Z^{a-2}$; ARA = $(1-a)/Z$

- $U(Z) = \ln(Z)$

we have $U' = 1/Z$; $U'' = -1/Z^2$; ARA $= 1/Z$

- $U(Z) = [1/(1-r)].Z^{1-r}$

we have $U' = Z^{-r}$; $U'' = -rZ^{-r-1}$; ARA $= r/Z$; ARR $= r$

Absolute risk aversion

$$ARA = -\frac{U''(Z)}{U'(Z)}$$

Relative risk aversion

$$ARR = -\frac{Z \,.\, U''(Z)}{U'(Z)}$$

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