Sequence 3 : Modelling risk and time

Unit 3 : Modelling time

Lesson 29 : Multi-period models

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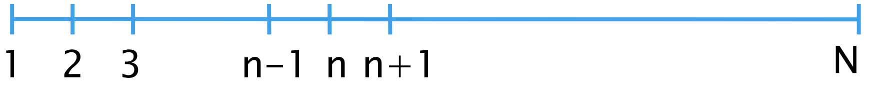
Multi-period models

- Activities and constraints -> for each period
- Inter-temporal optimization



N : planning horizon
Period during which decisions are made

- X_n level of activity X in year n
- Availability of resources in year n according to decisions made in n-1

=> Represent using transfer equation

The objective function

- According to results throughout entire planning horizon

- Aggregation updated over time
  -> updating rate
- If updating rate is unknown,
  use the long-term savings interest rate

The updating rate expresses the preferences of the decider for the present moment according to the future. The higher it is, the more the decider values current incomes over future ones.

- Maximization of updated available incomes (which can also be named consumption)

$$Max \cdot Z = \sum_{n=1}^{N} \frac{CONSO_n}{(1+a)^{n-1}}$$

$CONSO_n$: withdrawal for year n consumption
a : updating rate

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Constraints

- 2 types of constraints → The same as in a static model
  → Different because they link periods to one another

e.g. : land constraint with no land extension and/or annual rental
-> same as in static
land constraint with land purchase and/or rental over several years
-> constraints linked to one another

Livestock constraints

- Number of animals in year N may depend on year N-1
  - e.g. : transition link between 2 year old heifer/1year old heifer

$X_{heifer2,n} = X_{heifer1,n-1}$,
$X_{heifer2,n}$ number of 2 year old heifers in year n equals $X_{génisse1,n-1}$ number of 1 year old heifers in year n-1

- The farmer makes choices (herd extension, sales) which impact the years to come
  - e.g. : transition between cow and in-calf heifer (3 years) and cow of the following year

$X_{cow,N} = X_{cow,N-1} - AV_{cow,n-1} + X_{heifer3,N-1} - AV_{heifer3,n-1}$
$AV_{heifer3,n} \leq X_{heifer3,n}$

- Remember to take into account the initial situation otherwise the herd does not get larger !
  - e.g. : $X_{heifer2,n} = X_{heifer1,n-1} + X\text{-}init_{heifer1,n}$

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Investment constraints

- **Investment constraints**
  - An investment generates resources for the J years to come
  - $NI_n = \Sigma^J I_{n-(j-1)}$, $\forall$ n the investment level of year n depends on the investments of the j-1 previous years
  - Possibility of having NI_0

  - Investment financing :
    - Either annual depreciation -> cost in global margin
    - Or payment upon purchase

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Example of investment constraints

Example – model :

- 4 crops (C)
- 3 irrigation methods (R – without irrigation, surface, localised)
- 15 periods (T)
- Data concerning gross margins (MB), water needs (BE)
- Equipment for localised irrigation : lifespan of equipment : 5 years (J)

```
EQUATIONS        LAND(t)      land constraint
                 WATER           irrigation water constraint
                 EQUIPE(t)     drip equipment constraint
                 NEQUIP(t)     equipment level
                 MARGIN(t)      global margin calculation
                 OBJECTIVE      objective function ;
```

Availability constraint : same constraint as in the static model, indexed to set t (period)

Constraints that link years together

```
LAND(t)..    sum((c,r), X(c,r,t)) =l= sup ;
WATER(t)..      sum((c,r), be(c,r)*X(c,r,t)) =l= DE*SUP ;
EQUIPE(t)..  sum(c, X(c,'r2',t)) =l= NEQ(t);
```

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```
NEQUIP(t)      equipment level
MARGIN(t)      global margin calculation
OBJECTIVE      objective function              ;
```

```
NEQUIP(t)..  NEQ(t) =e= sum(j, AEQ(t-[ord(j)-1]));

MARGIN(t)..   MG(t) =e=  sum((c,r), mb(c,r)*X(c,r,t)) -
NEQ(t)*peq/card(j) ;

OBJECTIVE..   sum(t, MG(T)/((1+ta)**(ord(t)-1))) =e= Z ;
```

Cashflow constraints

- **Investment constraints**
  - Investment financing :
    - Either distribution of the annual depreciation -> weight in global margin
    - **Or payment upon the purchase of the investment**

$$IMMO_N = peq * AEQ_N$$

$$LIQ_{N-1} + LIQ0 + MG_N = IMMO_N + LIQ_N + CONS_N$$

LIQ_N	cashflow for year N
MG_N	global gross margin for year N
$IMMO_N$	money immobilized in investment
$CONS_N$	consumption withdrawal for year N
LIQ0	initial cashflow

It's your turn now !
Solution in the next video !

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